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Fuzzy Quasi Regular Ring

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Abstract - Let R be a commutative ring with unity. In this paper we introduce and study Fuzzy Quasi regular ring as generalizations of (ordinary) Quasi regular ring. We give some basic properties about these concepts.

Keywords: Fuzzy set ,Fuzzy ring , Quasi regular ring ,Fuzzy maximal ideal

I. INTRODUCTION

The notation of Fuzzy subset was introduced by L.A.Zadeh [1]. His seminar paper in 1965 has opened up new insights and applications in a wide range of scientific fields. Liu [2] has studies Fuzzy ideals and Fuzzy ring. Regular rings were presented by Skornyakov [3] as a generalization this to concept (Nicholson) gave the concept of Quasi regular ring [4]. We present Fuzzy Quasi regular ring by extending the notation of (ordinary) Quasi regular ring. We give some basic results about Fuzzy Quasi regular ring. Also we study the holomorphic image of Fuzzy Quasi regular ring.

II. FUZZY QUASI REGULAR RING

1.1 DEFINITION [1]

Let S be a non-empty set and I be the closed interval [0,1] of the real line (real numbers). A Fuzzy set A in S (a Fuzzy subset of S) is a function from S into I.

1.2 Definition [2]

Let X_t: S ⟶ [0, 1] be a Fuzzy set in S, where x∈S t∈[0,1] defined by: $X_t(y)$ =t if x=y, and $X_t(y) = 0$ if x≠y ∀y∈S. X_t is called a Fuzzy singleton or Fuzzy point on S.

1.3 Remark [1]

The following properties of level subsets hold for each t∈(0,1]

1. $(A\cap B)_t=A_t \cap B_t$
2. A=B if and only if $A_t=B_t$, for all t∈(0,1].

1.4 Definition [5]

Let (R,+,·) be a ring and let X be a Fuzzy set in ổ R. Then X is called a Fuzzy ring in ring (R,+,·) if and only if, for each x, y ∈ R

1. X(x+y) ≥ min{X(x), X(y)}
2. X(x) = X(– x)
3. X(xy) ≥ min{X(x), X(y)}.

1.5 Definition [6]

A Fuzzy subset X of a ring R is called a Fuzzy ideal of R, if for each x, y ∈ R

1. X(x–y) ≥ min{X(x), X(y)}
2. X(xy) ≥ max{X(x), X(y)}.

1.6 Definition [7]

A ring R is said to be Quasi regular if for $a \in R$ there is $e^2 = e \in Ra$ such that $a(1-e) \in J(R)$.

(For all $a \in R$, there is $x \in R$ such that $(xa)^2 = xa$ and $a - axa \in J(R)$).

We fuzzified this concept as follows.

1.7 Definition

Let X be Fuzzy ring, X is called Fuzzy Quasi regular

If and only if $\forall a_t \in X, \exists e_k \in X,\ e^2{}_k = e_k$

Such that $a_t(1_{x(0)} - e_k) \in F-J(X)$.

($\forall a_t \in X, \exists x_k \in X$ such that $a_t x_k = (a_t x_k)^2$,

$a_t - a^2{}_t x_k \in F-J(X)$)

1.8 Proposition

Let X be a Fuzzy ring $a_t \subseteq X, a_t \in F-J(X)$ then

$1 - a_t r_k$ invertible singleton in X.

Proof: Let $a_t \in F-J(X) \Rightarrow a_t \in$ any maximal Fuzzy ideal of X
ideal of X

Suppose $1_{x(0)} - a_t r_k$ is not invertible singleton, then there is a Maximal Fuzzy ideal A of X Such that $< 1_{x(0)} - a_t r_k > \subset A$.

Hence $1_{x(0)} - a_t r_k \in A$,but $a_t \in A$ implies that $a_t r_k \in A$, hence $1_{x(0)} \in A$.

This implies $A = X$ because for any $b_s \in X$, $b_s = 1_{x(0)} b_s$

Which implies $b_s \in A$ since $1_{x(0)} \in A$.

Thus $X = A$ which is contradiction.

1.9 Proposition

Let X be a Fuzzy ring then $(F-J(X))_t \subseteq J(X_t)$, $\forall t \in [0,1]$.

Proof: Let $a \in (F-J(X))_t$, Hence $a_t \in F-J(X)$

To prove $a \in J(X_t)$

We must prove $\forall r \in X_t$, $1 - ar$ invertible in X_t

Since $a_t \in X_t \Rightarrow r_t \in X$

Since $a_t \in F-J(X)$, $r_t \in X$, we have $1_{x(0)} - r_t a_t$ invertible (singleton) in X

Implies that $(1-ra)_t$ invertible singleton in X

Then $\exists b_l \in X$ such that $(1-ra)_t b_l = 1_{x(0)}$

Implies that $[(1-ra)b]_\lambda = 1_{x(0)}$

$\Rightarrow \lambda = x(0)$ and $(1-ra)b = 1$

$\Rightarrow (1-ra)$ inv.elem.in X_t

$\Rightarrow a \in J(X_t)$

The following result explains the relationship between Fuzzy Quasi regular ring and its level.

1.10 Corollary

Let X be a Fuzzy Quasi regular ring, then X_t is Quasi-regular ring, $\forall t \in [0,1)$.

Proof: Let $a \in X_t$, then $a_t \in X$ .But X is Fuzzy Quasi regular ring

So $\exists r_k \in X$ such that $a_t - a^2{}_t r_k \in F-J(X)$

Then $(a-a^2r)_\lambda \in F-J(X)$, $\lambda = \min\{t,k\}$

$a - a^2 r \in (F-J(X))_\lambda \subseteq J(X_\lambda)$,

Implies that $a - a^2 r \in J(X_\lambda)$

Since $\lambda = \min\{t,k\}$, so $\lambda = t$ or $\lambda = k$

If $\lambda = t$, then $a - a^2 r \in J(X_t)$ and X_t is quasi regular ring

If $\lambda = k$, then $\lambda < k$ and so $X_t \subseteq X_k = X_\lambda$

$\Rightarrow a - a^2 r \in J(X_t)$ and so X_t is Quasi regular ring

If $\lambda = k$,then k< t so $X_t \subseteq X_k \subseteq X_\lambda$

$\Rightarrow$ { maximal ideal in X_t } $\subseteq$ { maximal ideal in X_λ }

Thus $a - a^2 r \in J(X_t)$ and so X_t is Quasi regular ring

1.11 Proposition

X is Fuzzy regular ring if and only if X is Fuzzy Quasi regular ring and $F-J(X) = O_{x(0)}$

Proof: Let $a_t \in X$, since X is Fuzzy regular

$\Rightarrow \exists r_k \in X$ Suchthat

$a_t - a^2{}_t r_k = O_\lambda \subseteq O_{x(0)} \in F-J(X), \lambda = \min\{t,k\}$

Then X is Fuzzy Quasi regular ring

To prove $F-J(X) = O_{x(0)}$

Let $a_t \in F-J(X)$, since X is Fuzzy regular

$\Rightarrow \exists r_k \in X$ Such that $a_t - a_t^2 r_k \subseteq O_{x(0)}$

$\Rightarrow a_t(1_{x(0)} - a_t r_k) \subseteq O_{x(0)}$

By $(a_t \in F-J(X) \Rightarrow 1_{x(0)} - a_t r_k$ invertible)

$\Rightarrow b_l \in X$ such that $(1_{x(0)} - a_t r_k) b_l = 1_{x(0)}$

Then $(1_{x(0)} - a_t r_k) b_l \subseteq O_{x(0)}$

$a_t 1_{x(0)} \subseteq O_{x(0)} \Rightarrow a_t = O_t \subseteq O_{x(0)}$

Then $F-J(X) = O_{x(0)}$

1.12 Proposition

In a Fuzzy Quasi regular ring every Fuzzy non unit is a Fuzzy zero divisor

Proof: Let X is a Fuzzy Quasi regular ring then X_t is Quasi regular ring

Let $a_t \in X$ not unit, then $a \in X_t$ (a is non unit)

$\Rightarrow a - a^2 r \in J(X_t)$

$\Rightarrow 1 - a + a^2 r$ Invertible element

1.13 Theorem

Let X be a Fuzzy Quasi regular ring, then for every Fuzzy ideal of X

$I \cap K = I \circ K + F - J(x) \cap (I \cap K)$

Proof: Since $F-J(X) \cap (I \cap K) \subseteq I \cap K$

and $I \circ K \subseteq I \cap K$

$\Rightarrow I \circ K + F - J(X) \cap (I \cap K) \subseteq I \cap K$

Let $a_t \subseteq I \cap K$,since X is Fuzzy Quasi regular ring

$\Rightarrow \exists x_k \subseteq X$, $\ni a_t - a^2{}_t x_k \in F-J(X)$,

$a_t x_k = (a_t x_k)^2 \Rightarrow a_t - a^2{}_t x_k \in I \cap K$

$\Rightarrow a_t - a^2{}_t x_k \in F-J(X) \cap (I \cap K)$

Let $a_t - a^2{}_t x_k = b_l$, $b_l \in F - J(x) \cap (I \cap K)$

$\Rightarrow a_t = a^2{}_t x_k + b_l$, it is clear $a^2{}_t x_k \in I \circ K$

$\Rightarrow a_t \in I \circ K + F - J(X) \cap (I \cap K)$

$\Rightarrow I \cap K \subseteq I \circ K + F - J(X) \cap (I \cap K)$

1.14 Corollary

Let X be a Fuzzy Quasi regular ring, then for every fuzzy ideal of X

$I = I^2 + F - J(x) \cap I$

Proof: If $I = K$,then $K = K^2 + F - J(X) \cap K$

$I \cap K = (I \cap K)^2 + F - J(X) \cap (I \cap K)$,

$(I \cap K)^2 \subseteq I \circ K$

$\Rightarrow I \cap K \subseteq I \circ K + F - J(X) \cap (I \cap K)$

$I \circ K + F - J(X) \cap (I \cap K) \subseteq I \cap K$

$\Rightarrow I \cap K = I \circ K + F - J(X) \cap (I \cap K)$

Then $I = I^2 + F - J(x) \bigcap I$

1.15 Corollary

Let X be a Fuzzy Quasi regular ring then for every $a_t, b_h \subseteq X$

$$< b_l > \bigcap < a_t > = < (ab)_\lambda > + F - J(X) \bigcap < a_t > \bigcap < b_h >$$

1.16 Proposition

If A is Fuzzy ideal such that $A + < a_t > = X \quad \forall a_t \notin A$, then A is maximal Fuzzy ideal

Proof: Suppose $\exists B$ Fuzzy ideal such that $A \subset B \subseteq X$

Then $\exists a_t \in B$ and $a_t \notin A \Rightarrow A \subset A + < a_t > \subseteq B \subseteq X$

$\Rightarrow X \subseteq B \subseteq X \Rightarrow B = X$, then A is Fuzzy maximal

1.17 Lemma

Let $f : X \to Y$, f is homomorphism, then

$f(F - J(X)) \subseteq F - J(Y)$

Proof: $f(F - J(X)) \subseteq f(\bigcap_{i \in \Lambda} L_i)$

(L_l is Fuzzy maximal ideal in X $\subseteq \bigcap_{i \in \Lambda} f(L_i)$)

But $f(L_i)$ is Fuzzy maximal ideal in Y $\subseteq \bigcap F - J(Y)$

1.18 Proposition

Let $f : X \to Y$ epimorphism. If X is Quasi regular Fuzzy ring then Y is Quasi regular Fuzzy ring

Proof: Let $y_t \in Y$, since f is onto $\Rightarrow \exists x \in R$

such that $f(x) = y$, hence $y_t = f(x_t)$

But $f(x_t - x_t^2 r_k) \in F - J(X)$ for some $r_k \in X$

$\Rightarrow f(x_t - x_t^2 r_k) \in f(F - J(X)) \subseteq F - J(Y)$

$\Rightarrow f(x_t) - f(r_k)(f(x_t))^2 \in F - J(Y)$

$\Rightarrow y_t - r_t^/ y_t^2 \in F - J(Y)$

Then Y is Quasi regular Fuzzy ring

Recommendations

The extension to this can be given to find the Inverse Image for $f: X \to Y$ Quasi regular Fuzzy ring, Other researchers must conduct similar studies related to quasi regular Fuzzy ring and to further validate the claims of this study

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